Morphological properties of tunnel valleys of the southern sector of the Laurentide Ice Sheet and implications for their formation

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Abstract

Tunnel valleys have been widely reported on the bed of former ice sheets and are considered an important expression of subglacial meltwater drainage. Although known to have been cut by erosive meltwater flow, the water source and development of channels has been widely debated; ranging between outburst flood events through to gradually occurring channel propagation. We have mapped and analysed the spatial pattern and morphometry of tunnel valleys and associated glacial bedforms along the southern sector of the former Laurentide Ice Sheet from high-resolution digital elevation models. Around 2000 tunnel valleys have been mapped, revealing a well-organised pattern of sub-parallel, semi-regularly spaced valleys that cluster together in distinctive networks. The tunnel valleys are typically <20 km long, and 0.5-3 km wide and preferentially terminate at moraines. They tend to be associated with outwash fans, eskers, giant current ripples, and hill-hole-pairs. At the ice-sheet scale, we find most tunnel valleys occur on the flat portions of palaeo-ice sheet beds, where subglacial water flow would have been largely unconstrained by topography, while tunnel valley morphology is strongly modulated by local variations in basal conditions (e.g. thermal regime and topography) and hydrology (i.e. whether conduit erosion is up into the ice or down into the sediments). Analysis of cross-cutting relationships between tunnel valleys, moraines and outwash fans permits reconstruction of channel development in relation to the retreating ice margin. The reconstruction demonstrates incremental growth of valleys, with some used repeatedly, or for long periods, during deglaciation, while others were abandoned shortly after their formation. Our data and interpretation supports gradual (rather than a single-event) tunnel valley formation, with secondary contributions from flood drainage of subglacial and/or supraglacially stored water.

Key words: tunnel valleys; geomorphology; Laurentide Ice Sheet; subglacial meltwater; gradual or catastrophic

1. Introduction

Incised into bedrock or sediment, tunnel valleys and channels (hereafter referred together as tunnel valleys) are elongate depressions up to several kilometres wide, with undulating long-profiles, tens of kilometres long and tens to hundreds of metres deep. They are observed in many formerly glaciated
landscapes around the world, and tend to be orientated parallel to the direction of former ice flow (e.g. Wright, 1973; Attig et al., 1989; Wingfield, 1990; Piotrowski, 1994; Patterson, 1997; Huuse & Lykke-Anderson, 2000; Jørgensen & Sandersen, 2006). Features with similar dimensions have also been described beneath current ice masses (e.g. Rose et al., 2014). Tunnel valley formation is typically attributed to subglacial meltwater erosion at the base of ice sheets (cf. Ó Cofaigh et al., 1996; Kelew et al., 2012; van der Vegt et al., 2012), and they are considered an important component of the subglacial hydrological system, providing drainage routeways for large volumes of water and sediment. Understanding their genesis is relevant for reconstructing former ice sheets, elucidating basal processes and exploiting the geomorphological record in a way that is useful for modelling subglacial hydrology. However, despite being debated for over 100 years, there is considerable uncertainty about the underlying processes governing tunnel valley formation. This debate is focused around two genetic models: ‘outburst’ formation and ‘gradual or steady-state’ formation (Fig. 1) (cf. Ó Cofaigh et al., 1996; Kelew et al., 2012; van der Vegt et al., 2012).

The ‘outburst’ hypothesis (Fig. 1a) ascribes the erosion of tunnel valleys to rapid drainage of sub- or supraglacially stored meltwater. Contemporary observations from the Antarctic and Greenland ice sheets demonstrate the efficacy of meltwater storage and drainage in sub- and supraglacial environments (Zwally et al., 2002; Wingham et al., 2006; Fricker et al., 2007; Das et al., 2008) and it is reasonable to expect that the Laurentide Ice Sheet experienced similar events. In addition, the impoundment of meltwater behind a frozen ice margin has been linked to tunnel valley formation, for example, along the southern terrestrial margins of the former Laurentide and European ice sheets where permafrost was prevalent (e.g. Piotrowski, 1994; Cutler et al., 2002; Hooke & Jennings, 2006).

Genesis is typically thought to occur via repeated low to moderate magnitude floods that may be at or below bankfull flow (e.g. Wright, 1973; Boyd, 1988; Wingfield, 1990; Piotrowski, 1994; Cutler et al., 2002; Hooke & Jennings, 2006; Jørgensen & Sandersen, 2006). Catastrophic erosion of entire tunnel valley networks by massive sheet floods (bankfull flow) has also been proposed (e.g. Shaw & Gilbert, 1990; Brennand & Shaw, 1994, Shaw, 2002), but has been considered less likely given the very large volumes of stored water required (e.g. Ó Cofaigh et al., 1996; Clarke et al., 2005).

The ‘gradual’ or ‘steady-state’ hypothesis (Fig. 1b) typically invokes erosion of soft-sediment beds in low pressure subglacial channels (Boulton & Hindmarsh, 1987; Mooers, 1989; Praeg, 2003; Boulton et al., 2009). In this model, high water pressures transmitted through the substrate to the ice-sheet terminus initiates failure and headward erosion of a conduit (by piping) (Shoemaker, 1986; Boulton & Hindmarsh, 1987; Hooke & Jennings, 2006; Boulton, 2009). As the fluid pressure of the conduit is lower than the surrounding substrate, meltwater flows towards the conduit, the walls are enlarged by sapping (i.e. undermining and headward recession of a scarp) and the sediments are mobilized and transported away by the resulting subglacial stream (Boulton & Hindmarsh, 1987). In general, enlargement is suggested to occur via steady-state Darcian flow of water into the conduit (e.g.
Boulton & Hindmarsh, 1987; Boulton et al., 2007a,b, 2009). Hooke & Jennings (2006) adapted this hypothesis, suggesting that initial headward erosion by piping was followed by more rapid enlargement when the conduit tapped into a subglacial lake, thereby combining both scenarios in Figure 1. Ravier et al. (2014) emphasised the potential influence of localised high porewater pressures in promoting efficient erosion by hydrofracturing and brecciation, while Mooers (1989) considered supraglacial drainage to the bed rather than basal meltwater as the dominant source for gradual tunnel valley erosion.

A range of approaches can be applied to the investigation of tunnel valleys including theoretical, sedimentological and morphological. Thus far, most effort has used a combination of these approaches, with much data, description and detail, but for a small number of tunnel valleys (see Section 2). From these it is difficult to extract representative information of the population of tunnel valleys or to gain an understanding of the broader-scale distribution of landforms. To rectify this we undertake a systematic and large-scale mapping campaign of the size, shape, pattern and distribution of tunnel valleys to better understand the spatial properties of this phenomenon, noting that it is useful to know more precisely what it is that requires explanation (e.g. Dunlop & Clark, 2006, for ribbed moraine). In doing so we will answer the following questions: (1) what constitutes a tunnel valley and how can they be distinguished in the geological record? (2) What are the morphological characteristics of a tunnel valley? (3) Is there a characteristic distribution and network arrangement? (4) Are there systematic associations between tunnel valleys and other landforms? The southern sector of the Laurentide Ice Sheet was selected because it contains thousands of these landforms, they can be identified from digital elevation models (DEMs) and the distinctive geometry of the ice lobes provides information on the water drainage pathways. Our mapping builds on and replicates, in many places, comprehensive local and regional studies, which include sedimentological details that we draw on. Our data provide basic metrics on tunnel valleys and their variation in scale and pattern and should promote new insights into tunnel valley formation and meltwater drainage and erosion beneath ice sheets.

Limitations

The partial or complete burial of tunnel valleys by glacial and post-glacial infill (cf. Kehew et al., 2012; van der Vegt et al., 2012) limits the mapping and measurement of tunnel valleys from DEMs. Buried tunnel valleys with no surface expression cannot be identified by the mapping. However, buried valleys are rare or at least fewer have been identified along the southern margin of the Laurentide Ice Sheet compared to the European ice sheets (e.g. Jørgensen & Sandersen, 2006; Kristensen et al., 2007, 2008; Stewart & Lonergan, 2011). Moreover, unless there is a systematic bias predisposing burial in some locations over others then the mapped pattern and distribution of tunnel valleys is likely to be informative.
2. **Previous work and observations in study area**

There is a rich history of work on tunnel valleys beneath the southern margin of the former Laurentide Ice Sheet (Wright, 1973; Attig et al., 1989; Mooers, 1989; Patterson, 1994, 1997; Clayton et al., 1999; Johnson, 1999; Kehew et al., 1999, 2013; Cutler et al., 2002; Sjogren et al., 2002; Fisher et al., 2005; Kozlowski et al., 2005; Jennings, 2006; Hooke & Jennings, 2006; Kehew & Kozlowski, 2007). In this section we briefly summarise key observations arising from this work, which need to be incorporated into any model of tunnel valley formation.

Tunnel valleys are commonly observed on the bed of the former southern margin of the Laurentide Ice Sheet and typically occur as distinct radiating sets of regularly spaced valleys associated with eskers and terminal or recessional moraines (cf. Kehew et al., 2012). At the bed of the Saginaw Lobe, for instance, valleys are typically spaced at 6-10 km intervals (Fisher et al., 2005; Kehew et al., 2013). Tunnel valleys are incised into glacial sediments up to a depth of 25 m and extend for <50 km (e.g. Jennings, 2006). However, tunnel valleys up to 150 km long have been documented in the Superior Lobe, Minnesota (Wright, 1973), and valleys are eroded up to 200 m into the bedrock floors of Lake Superior and Lake Michigan (Regis, 2003; Jennings, 2006).

Although tunnel valleys are typically sub-parallel, they are also observed to join, split and even cross-cut each other (e.g. Wright, 1973; Mooers, 1989; Kehew et al., 1999, 2005; Fisher et al., 2005; Kehew & Kozlowski, 2007). Cross-cutting relationships, both between tunnel valleys and with other glacial landforms (e.g. drumlins, outwash fans, moraines), record a palimpsest signature of tunnel valley erosion. In the Saginaw Lobe, Kehew et al. (1999, 2005) and Kehew & Kozlowski (2007) identified a series of palimpsest associations in which partially buried tunnel valleys pass beneath terminal moraines, diamicton and surficial outwash associated with later advances. This palimpsest style is interpreted to result from the collapse of ice and debris into the valley, which becomes (partially) buried by sediment during a re-advance and then re-emerges as the ice melts out (e.g. Kehew & Kozlowski, 2007).

Tunnel valley morphology ranges from sharply-defined with constant or downstream increasing dimensions (e.g. Mooers, 1989), to indistinct valleys often associated with hummocky terrain and characterised by beaded or crenulated planforms, or as a series of aligned depressions (e.g. Kehew et al., 1999; Sjorgen et al., 2002). Indistinct valleys may be due to partial burial during re-advance events or by melt out of debris rich ice obscuring them (Kehew et al., 1999). Sjorgen et al. (2002) also identified indistinct valleys in Michigan that are eroded into the hummocky terrain.

In Wisconsin, Michigan and Minnesota, bands of hills are observed to occur upstream of tunnel valleys (Johnson, 1999). These are interpreted as erosional remnants of an anastomosing subglacial meltwater system that drained along the inter-hill valleys. At their downstream end, tunnel valleys...
often terminate at outwash fans (e.g. Attig et al., 1989; Mooers, 1989; Patterson, 1994; Clayton et al., 1999; Johnson, 1999; Kehee et al., 1999; Derouin, 2008), some of which contain coarse boulder-gravel material, which is interpreted as evidence for outburst events (Cutler et al., 2002).

3. Methods

3.1 Datasets and mapping

For this study, we used the National Elevation Dataset (NED) (http://nationalmap.gov/elevation.html), which is a seamless DEM with a resolution of 1/3 arc seconds (~10 m) across the entire study area, and 1/9 arc seconds (~3 m) in some locations. Surficial and bedrock geology maps (e.g. Fullerton et al., 2003; Soller et al., 2011) were also used to aid identification and interpretation. Glacial landforms were identified according to conventional criteria and digitised directly into a Geographical Information System (GIS). Polylines were used to map tunnel valleys sides and centrelines, eskers and moraines. Polygons were used to map hill-hole pairs, outwash fans and dissected hills.

3.2 How do we distinguish tunnel valleys in the geological record?

Apart from tunnel valleys, large elongate depressions with similar dimensions may also form by fluvial erosion (river valleys), proglacial meltwater erosion (spillways), subglacial abrasion/plucking (overdeepenings), or arise from geological structures (e.g. fault lines). These phenomena are readily observed today and the formative mechanisms are reasonably well known. In contrast, tunnel valleys have not been observed actively forming beneath, or at the margins of, modern day ice sheets, and so their genesis and properties are more enigmatic. In the geological record they have been distinguished by their large size and characteristics such as their orientation parallel to inferred ice flow, undulating thalwegs and associations with subglacial bedforms and eskers; all pointing to a subglacial origin. In particular, undulating thalwegs and their association with eskers and outwash fans, permit them to be distinguished from proglacial and fluvial rivers. However, negative evidence (e.g. no esker found in a valley) does not necessarily preclude a subglacial origin, and it is not known whether size is actually a distinguishing feature or if, for instance, much smaller meltwater channels (tens of metres in width; e.g. Greenwood et al., 2007) are less mature forms of a continuum of glacial hydrological channels.

For the purposes of this study we restrict our definition of a tunnel valley to subglacially eroded channel-forms. Tunnel valleys that could clearly be differentiated as being eroded into bedrock were not mapped as their formation is more difficult to decipher from geological structures or glacial overdeepenings and valleys abraded and plucked by overlying ice. All potential tunnel valleys were mapped and then assessed to determine whether they formed subglacially. To determine whether valley thalwegs are undulating the number of negative and positive slope segments over 100 m length scale were calculated. Each valley was then assigned a confidence level from one to three, with one
being the most certain and three the least (Fig. 2). Channels lacking undulations and that do not contain subglacial bedforms are difficult to differentiate from proglacial or postglacial channel systems and were therefore given a confidence of 3. Valleys with an undulating long-profile, which contain eskers or terminate in outwash fans were classified as ‘certain’ tunnel valleys and given a confidence level of one (Fig. 2a-d). Only those tunnel valleys with a confidence level of one or two were used in the spatial and morphological analyses.

3.3 Tunnel valley measurements

Using the centrelines of tunnel valleys, we computed their length. Where two or more tributaries coalesce, the longest routeway was used to determine length. Tunnel valley width (distance between mapped valley sides) was measured from cross-profile transects positioned at 1 km intervals along the centreline of each tunnel valley. The influence of local elevation gradient ($G_{loc}$) on along valley changes in width ($W_{loc}$) was calculated at each 1 km interval ($j$) using equations [1] and [2]:

$$G_{loc} = \frac{(E_{j+1} - E_{j})}{(D_{j+1} - D_{j})} \quad [1]$$

$$W_{loc} = \frac{(W_{j+1} - W_{j})}{(D_{j+1} - D_{j})} \quad [2]$$

Where $E = \text{elevation}$, $W = \text{width}$ and $D = \text{downstream distance}$. To calculate tunnel valley spacing it was necessary to restrict our analysis to networks comprising distinct populations of similar orientation, which were likely formed during a similar drainage phase. We calculated the spacing of 966 tunnel valleys organised in 24 discrete networks. Spacing ($S$) was calculated from cross-profile transects orientated perpendicular to the direction of the network and positioned at 5 km intervals along each long profile. A median spacing value and the standard deviation ($\sigma$) was calculated for each drainage network. To provide an indication of tunnel valley regularity per network the coefficient of variation ($\sigma / \text{mean spacing}$), expressed as a percentage ($\sigma\%$), was also calculated (Hovius, 1996; Talling et al., 1997); tunnel valley networks with a low $\sigma\%$ exhibit low variability in spacing.

To investigate drainage evolution during deglaciation, a subset of meltwater features were grouped into ‘drainage-sets’, defined as a collection of features that formed during the same drainage phase. This was based on cross-cutting relationships (e.g. between channels, outwash fans and moraines) to reconstruct a relative history of drainage activity. Cross-cutting relationships between tunnel valleys and moraines were classified according to whether the tunnel valley: (1) terminates at a moraine at its downstream end and therefore formed contemporaneously with it; (2) is overlain by moraines along its length, thus suggesting that the tunnel valley was no longer active when the moraines were deposited; or (3) breaches moraines along its length, thereby indicating that the tunnel valley...
continued to drain water, either destroying pre-existing or preventing moraines from forming during
retreat.

4. Properties of Tunnel Valleys

4.1 Is there a characteristic distribution and network arrangement?

4.1.1 Distribution

Figure 3 shows the distribution of all 1931 tunnel valleys (1694 of which have a confidence of 1 or 2)
mapped beneath the terrestrial southern sector of the Laurentide Sheet. We estimate that ~80% of
these tunnel valleys have been previously identified and mapped during more localised investigations
(e.g. Wright, 1973; Attig et al., 1989; Patterson, 1997; Fisher et al., 2005). The map reveals a
tendency for tunnel valleys to cluster together in distinctive ‘networks’ with large intervening areas
where no or very few valleys occur. Networks mostly avoid running down the central axes of major
ice lobes. They are instead concentrated along suture zones between adjacent ice lobes or at the edge
of linear to slightly lobate ice-margin positions. Tunnel valleys are rarer and more dispersed or
isolated at the southernmost (LGM) margins of the James, Des Moines, Lake Michigan and Erie-
Huron ice lobes (Fig. 3). Those that do occur in these ice lobes tend to be positioned up-ice, either at
the lateral margins of the LGM lobes (e.g. Green Bay Ice Lobe) or at recessional moraines (e.g. Des
Moines Ice Lobe).

Tunnel valley networks often occur down ice-flow of basins or sub-basins (Fig. 3). For example, the
Saginaw Lobe tunnel valley network emanates from an arm of the present-day Lake Huron Basin, the
Langlade and Chippewa tunnel valley networks are all associated with sub-basins of the present-day
Lake Superior, and tunnel valleys occur downstream of the low-relief trough of the Des Moines Lobe.
Based on modelled hydraulic potential surfaces, Livingstone et al. (2013) predicted that the Lake
Superior Basin and NE sector of the Lake Michigan Basin were sites of several subglacial lakes
during the last glacial (marked in Fig. 3). There appears to be no clear link between these lake and
tunnel valleys. On the other hand, subglacial lakes may also have been present elsewhere in the Great
Lake Basins and it is noteworthy that tunnel valleys are commonly downstream of these basins.

4.1.2 Network arrangement

The overall shape of tunnel valley networks varies (Fig. 3), with both broad networks composed of
many short valleys (e.g. Green Bay, James and SE edge of Superior), and narrow networks composed
of long valleys (e.g. Superior, Huron-Erie and Langlade). Cross-cutting of tunnel valleys occurs both
between and within networks.
Overall tunnel valley spacing (Fig. 4) displays a positively skewed, unimodal distribution with a median spacing of 4.5 km and standard deviation of 4.6 km (\(\sigma% = 81\)). However, the median spacing of individual tunnel valley networks ranges from 1.9 to 9.1 km. Tunnel valleys in the Green Bay (median: 2.9 km), Superior (median: 3.7 km) and Huron-Erie (median: 1.9 km) lobes are closely spaced. Conversely, tunnel valley networks in the large Saginaw (median: 5.7 km), Michigan (median: 5.5 km) and Des Moines (median: 5.4 km) lobes and in North Dakota (median: 5.1 km) have a wider than average spacing. In all of the measured networks the standard deviation of the tunnel valley spacings is less than the mean tunnel valley spacing, and 9 of the 24 networks are <60\%.

There is no significant correlation between the number of tunnel valleys within a network (ranging from 7 to 249) and the standard deviation, but the standard deviation increases as the mean and median network spacing increases, hence the use of the coefficient of variation (\(\sigma%\)).

4.2 What are the morphological characteristics of a tunnel valley?

The lengths of mapped tunnel valleys display a unimodal, positively skewed distribution, which is approximately log-normal (Fig. 5a). Lengths range from 200 m to 65 km, with a mode of 7-9 km, median of 6.4 km and standard deviation of 8 km. Long and short tunnel valleys occur in most places, although long valleys are less common in the Green Bay and Huron Erie lobes, and dominate in the Superior, Langlade, Wadena, Michigan and Saginaw lobes.

The widths of mapped tunnel valleys display a unimodal distribution with a positive skew, which approximates normal when log-transformed (Fig. 5b). Tunnel valley widths vary considerably across the study area, ranging from 15 m to 6.7 km, with a mode of 600-800 m, median of 550 m and standard deviation of 660 m. The Chippewa, Langlade and Michigan valleys are consistently wide (typically >600 m), while the Huron-Erie, Superior, Green Bay and Des Moines valleys are narrow (<600 m). Other networks, in the Saginaw, Superior and Wadena lobes, comprise a mix of wide and narrow valleys. There is a tendency for longer tunnel valleys to be wider (power law function, \(r^2 = 0.38, p\text{-value} = <0.001\)) (Fig. 6).

Tunnel valley planform shape varies across the study area (Fig. 7). The majority consist of a single valley ‘thread’; more than two orders of ‘stream ordering’ are rare and tributaries tend to be restricted towards valley heads (Figs. 2, 3, 7). Valley margins range from sharp to indistinct and from crenulated to straight. Straight margins are more typical of long, thin tunnel valleys (Fig. 7a,d,f). However, many margins are crenulated, with bulbous and abrupt angular morphologies that result in large down-valley changes in width (Fig. 7a-f). Figure 8 demonstrates a weak relationship between tunnel valley width and distance downstream. Valleys both widen and narrow downstream with considerable and abrupt variations in width. The variation in tunnel valley width bears no relation to the local elevation gradient (Fig. 9). Local along-valley elevation gradients are relatively low (typically <\(\pm 1.5\)) and valleys widen and narrow on both reverse and normal slopes.
Tunnel valleys and tunnel valley segments often start and end abruptly and can appear fragmented or contain bulbous depressions (Fig. 7). The gaps between segments of tunnel valleys may show no evidence of modification (Fig. 7e,f); are partially incised by narrower and more discontinuous valleys or sets of parallel valleys (Fig. 7e); or consist of a series of depressions and hummocks with indistinct valley planform (Fig. 7a,b,d). The up-glacier ends of tunnel valleys range from rounded heads with steep sides (amphitheatre) (Fig. 7a,f) to open or indistinct (Fig. 2c-d). In Figure 7e-f, tunnel valleys comprise parallel tracks of two or more tightly spaced (<1 km) valleys.

4.3 Are there systematic associations between tunnel valleys and other landforms?

4.3.1 Moraines

The association between moraines and tunnel valleys varies with some valleys cutting through moraines (Fig. 10a); while in other locations moraines are superimposed on the valley or the valley terminates at a moraine (Fig. 10b). In Figure 10a, tunnel valleys cutting through an end moraine are observed to narrow and then trend down-glacier into esker and outwash fan deposits. Up-glacier of the end moraine are low relief (1-2 m) and regularly spaced transverse ridges (‘washboard’ moraine). They have a cuspathe geometry with the horns pointing up-glacier and converging on tunnel valley positions (see also Stewart et al., 1988; Cline et al., 2015). Fig. 10b shows examples of tunnel valleys terminating at, cutting through and overlain by recessional moraine. The tunnel valley network does not show a consistent pattern, with neighbouring channels exhibiting different moraine associations. Some valleys are continuous or semi-continuous, with a single outwash fan at, or just down-glacier from the terminus, and a series of on-lapping recessional moraines up-glacier. Elsewhere, valleys contain multiple outwash fans deposited at successive moraine positions.

4.3.2 Hill-hole pairs

We mapped 12 hill-hole-pairs (Bluemle and Clayton, 1984), 11 of which are found in North Dakota. Typically, hill-hole pairs comprise isolated features, but 4 of them are associated with tunnel valleys (e.g. Figs. 2c, 11). These seem to occur at the down-glacier end of the valleys, with smaller channels and eskers emanating from and diverging around the ice-thrust hill (Fig. 11a,b). In Fig. 11a, an esker emanating from one of the hill-hole pairs trends into another tunnel valley segment further down-glacier.

4.3.3 Outwash Fans

We mapped 187 outwash fans across the study area, predominantly at the downstream end of, but also within and between segments of tunnel valleys at moraine positions (Fig. 10b). Many of the outwash fans are connected upstream to an esker. Multiple trains of outwash fans occur along some tunnel valleys, but not all tunnel valleys are associated with outwash fans.
4.3.4 Giant Current Ripples

In Minnesota the floor of one tunnel valley is shown to contain regularly spaced sinusoidal bedforms orientated roughly perpendicular to the valley long profile (Fig. 12). The bedforms are 0.2-1.9 m high (H), 10-60 m long (L) and their crests are straight to slightly sinuous. Our data show that longer bedforms tend to be higher (linear regression, \( r^2 = 0.5 \)), and that the H:L ratio is ~0.02. The tunnel valley that the bedforms are constrained within is partially incised into underlying drumlins orientated obliquely to the valley long axis. An esker running NW-SE is overprinted on the bedforms. The southern end of the valley is bisected by a large (1 km diameter) circular incision with an intact central island.

The dimensions and shape of the transverse sinusoidal bedforms, the tendency for longer bedforms to be higher and their association with the tunnel valley is consistent with giant current ripples (e.g. Bretz et al., 1956; Carling, 1996; Rudoy, 2005). Given the undulating valley thalweg and superimposition of an esker on top of the ripple-forms, we suggest the simplest explanation is that the valley, circular incision and ripples were formed subglacially.

5. Discussion

5.1 Distribution and pattern of tunnel valleys

5.1.1 Southern sector of the former Laurentide Ice Sheet

The large-scale distribution of tunnel valleys is strongly controlled by ice geometry. Tunnel valleys are rare or absent at the terminus of major ice lobes, particularly those that are long and thin (e.g. James and Des Moines lobes), and are more common in interlobate regions, at the side of lobes or where the lobe exhibits a broader geometry (Fig. 3). This is consistent with theoretical drainage of meltwater beneath an ice lobe, which is strongly controlled by the ice-surface slope (e.g. Shoemaker, 1999). Meltwater is theorised to radiate out from the centre of lobes, and converge along interlobate regions where the subglacial hydraulic gradient and ice surface are relatively steep (Fig. 13). Indeed, tunnel valley networks associated with lobate margins often have a distinctive divergent geometry (Fig. 3).

The locations of ice lobes along the southern sector of the Laurentide Ice Sheet are topographically controlled and are inferred to have been fast-flowing (e.g. Mickelson and Colgan, 2003; Margold et al., 2015). Fast ice-flow is likely to have been promoted by thermomechanical feedbacks, enhancing basal meltwater production and lubricating the bed (cf. Winsborrow et al., 2010 and references therein). It is therefore no surprise that tunnel valleys are typically found down-glacier of basins, where the greatest volumes of basal meltwater were focused. However, there is no clear link to predicted subglacial lake locations or with their obvious drainage corridors except for the Langlade.
Lobe tunnel valley network (Fig. 3) (Livingstone et al., 2013). This suggests that the drainage of subglacially stored water was not the main control on tunnel valley formation, or that we have yet to discover the true extent of subglacial lakes (i.e. the prediction in Fig. 3 is an underestimate). For example, the predictions do not account for the possibility of water ponding behind frozen margins as suggested by Cutler et al., (2002) and Hooke & Jennings, (2006).

Measurements of tunnel valley spacing reveal an overall median spacing of 4.5 km with some degree of intra-network regularity (Fig. 4). Inter-network variation is greater, with median network values ranging from 1.9 to 9.1 km across the study area. The spacing metrics are within the range of previously reported values for tunnel valleys (Praeg, 2003; Jørgensen and Sandersen, 2009; Stackebrandt, 2009; Moreau et al., 2012; Kehew et al., 2013) but smaller than the average spacing of eskers (Storrar et al., 2014a, and references therein). Theory suggests that the spacing of subglacial conduits is controlled by substrate properties, basal melt rate and the hydraulic potential gradient (e.g. Boulton et al., 2007a,b, 2009; Hewitt, 2011). According to such theory the spacing between adjacent tunnel valleys should be wider if: (i) bed transmissivity is larger; (ii) melt rate/discharge is lower; and/or (iii) the subglacial hydraulic gradient is smaller. Thus the wider than average spacing towards the terminus of major ice lobes where ice surface slopes and thus hydraulic gradients are inferred to be shallower (e.g. Des Moines and Saginaw lobes – Clark, 1992), and a smaller spacing along narrow ice lobes characterised by steeper ice-surface and hydraulic gradients (e.g. Green Bay and Superior lobes – Clark, 1992) is consistent with theory. However, cross-cutting relationships indicate that not all tunnel valleys were acting synchronously, even within a drainage network (Fig. 10b), which might explain the large variations in spacing.

5.1.2 Geographical distribution of tunnel valleys during the last glaciation

Figure 14 displays the geographical distribution of tunnel valleys reported in the northern hemisphere and attributed to the last glaciation. It appears that tunnel valleys tend to be associated with the flat southern margins of terrestrial or formerly terrestrial (e.g. North Sea) palaeo-ice sheets. They also tend to occur towards the maximum limit of glaciation and are often found downstream of large basins such as the Witch Ground in the North Sea, Baltic Depression along the southern limit of the European Ice Sheet, and Great Lake basins along the southern limit of the Laurentide Ice Sheet.

The tendency for tunnel valleys to form on beds of low relief and gradient implies a genetic association. In particular, water flow in regions of low bed relief is largely unconstrained by topography and can therefore more easily erode laterally producing wide, shallow valley geometries. Conversely, more rugged terrain will exert a greater control on water flow, increasing network complexity and restricting valley expansion. A consequence of ice lobes along the southern margin of the Laurentide Ice Sheet having such shallow ice-surface slopes (reconstructed as 0.001 to 0.005
m/km Wright, 1973; Mathews, 1974; Clark, 1992), is the resulting low subglacial hydraulic gradients. Such low gradients are at odds with the development of many closely spaced large channels (Hewitt, 2011). This could indicate either that: (i) large discharges of subglacial meltwater were needed to form the tunnel valleys; or (ii) tunnel valleys and their spacing were determined by initial conditions set up near the ice margin (i.e. where ice-surface slopes are steepest and the greatest volumes of meltwater are discharged). Certainly, shallow ice-surface slopes would have extended the size of the ablation zone and made it more sensitive to small changes in summer air temperature, while hydrofracture of surface meltwater to the bed is easier where ice is thin.

The prevalence of tunnel valleys along terrestrial margins hints at an important role of permafrost in their formation (e.g. Wright, 1973; Piotrowski, 1994, 1997; Cutler et al., 2002; Jørgensen & Sanderson, 2006). It has been proposed that the development of a frozen toe along the fringe of an ice sheet acted as a barrier to water flow facilitating tunnel valley formation by subglacial ponding and outburst cycles (e.g. Wingfield, 1990; Piotrowski, 1994, 1997; Cutler et al., 2002). Moreover, freezing of sediment deposited in channels under the thin fringe of the ice sheet during winter months may have helped to prevent creep-closure of incipient tunnel valleys, thereby stabilizing and preserving their forms from year to year.

The occurrence of tunnel valleys near the LGM limit could indicate larger subglacial meltwater fluxes concomitant with greater catchment areas, a climatic control and or variations in basal conditions. Conversely, the paucity of tunnel valleys towards the centre of former ice sheets suggests formation is not linked to greater volumes of supraglacial meltwater production concomitant with climatic warming, although this may be partially counteracted by reduced erosion on the hard crystalline bedrock towards the centre of the Northern Hemisphere palaeo-ice sheets (Clark and Walder, 1994). Critically, the northern hemisphere Quaternary ice sheets were vastly different sizes, so it seems unlikely that tunnel valley distribution was a function of subglacial hydrological catchment size and meltwater flux, particular as the hydrological budget is likely to be dominated by supraglacial meltwater inputs during deglaciation. The width of the frozen toe is likely to decrease during retreat because adjustment of the thermal structure of the toe will lag considerably behind adjustment of the margin position to an ameliorating climate. Decrease in tunnel valley occurrence away from the maximum ice limit may therefore be indicative of a change to temperate glacier conditions.

5.2 Morphology of tunnel valleys

The tunnel valleys extend for up to 55 km, although the majority (90%) are <17 km long and the median is 6.4 km (Fig. 5a). In comparison, reported tunnel valley lengths from the North Sea range from a few kilometres to around 100 km, with the length of individual segments not normally exceeding 20-30 km (e.g. Huuse and Lykke-Andersen, 2000). Although very wide tunnel valleys were
found (maximum width ~6.7 km), the majority (90%) are 500-3000 m (Fig. 5b). This is similar to tunnel valley widths (500-5000 m) reported in Europe and elsewhere in North America (e.g. Brennand and Shaw, 1994; Huuse and Lykke-Andersen, 2000; Jørgensen and Sandersen, 2006; Kristensen et al., 2007).

Tunnel valley length and width display log-normal distributions (Fig. 5), which is common of other glacial landforms (Fowler et al., 2013; Hillier et al., 2013; Spagnolo et al., 2014; Storrar et al., 2014). Log-normal distributions are thought to typically emerge from many independent random events in which incremental growth or fragmentation occurs (e.g. Limpert et al., 2001). For drumlins and MSGLs a log-normal distribution has been used to suggest a growing phenomenon that occurs randomly, for random durations, or under random conditions (Hillier et al., 2013; Spagnolo et al., 2014), while for eskers it is thought to reflect ridge fragmentation (Storrar et al., 2014a). Examples of aligned tunnel valley segments characterised by abrupt start and end points implies at least some tunnel valley fragmentation, and this may occur due to partial burial during re-advance events or the melt out of debris-rich ice (Kehew et al., 1999), or differential erosion along the length of a drainage route (Fig. 7a,e,f; see also Sjorgen et al., 2002). However, in other cases aligned tunnel valley segments could indicate a time-transgressive origin (e.g. Mooers, 1989; Patterson, 1994; Jørgensen and Sandersen, 2006; Janszen et al., 2012). This is particularly apparent where the valley segments terminate in outwash fans, and/or where segments cross-cut each other (Fig. 10 and see also Mooers, 1989). The positive relationship between tunnel valley length and width (Fig. 6) is consistent with a growing phenomenon (e.g. by headward expansion) or continuous flow (e.g. a river). In contrast, the length and width of valleys formed by floods are likely to be independent of each other; length is related to the distance that the stored water body is from the ice margin, while width is a function of the magnitude and/or frequency of drainage.

In fluvial geomorphology, channel width in an equilibrium system increases downstream (Fig. 7f) and has classically been related to discharge, and hence drainage area (Leopold and Maddock, 1953; Leopold et al., 1964). This may be complicated locally by the erodibility of the bed substrate and channel slope (e.g. Finnegan et al., 2005). In contrast, large single source flood events (as may occur during a subglacial or supraglacial lake drainage event), will produce a relatively constant channel width (e.g. Lamb and Fonstad, 2010), or even show a downstream decrease if infiltration is significant (Fig. 7f). The downstream width of tunnel valleys in our dataset varies considerably and there is no systematic downstream trend in valley form, although general increases and decreases in width do occur (Figs. 6, 7a-e). Thus, there is no observable signature of catastrophic (constant or declining width) or stable, bankfull drainage (steady widening). Moreover, the downstream variation in widths is also inconsistent with subglacial drainage channels fed by multiple supraglacial lake inputs (e.g. Palmer et al., 2011), which we would expect to produce a downstream increase in width concomitant with increased water added.
Figure 7a-e indicates that local variations in tunnel valley width are generally more pronounced than any downstream trend. These widening’s could arise from basal conditions at the time of formation (e.g. thermal regime), catastrophic drainage (e.g. Sjorgen et al., 2002), or a laterally migrating stream at the base of the valley floor. Laterally migrating streams are unlikely as we do not observe terraces, bars or incised braided or meandering channels within the broader tunnel valleys, although this may partially be due to ice and post-glacial modification. The crenulated margins, circular incisions, residual hills, hummocky terrain and valley discontinuities are all analogous to features eroded during large floods by macroturbulent flow (e.g. Sjorgen et al., 2002), although these are typically associated with bedrock channels (Baker, 2009 and references therein). Moreover, we see little evidence for other characteristic features, such as irregular anabranching channels (although they are observed elsewhere, e.g. Boyd, 1988; Brennand and Shaw, 1994), inner channels, furrows and large bars (e.g. Channeled Scablands: Bretz, 1923), while residual hills are not typically streamlined.

The alternative to the catastrophic hypothesis is that variations in width are strongly controlled by local basal and hydrological conditions. Indeed, there is greater similarity between tunnel valleys from the same network (e.g. in form, size and association with other landforms) compared to tunnels valleys from different networks, which hints at the importance of local conditions. Although there is no clear association with bed slope (Fig. 9) or geology, the strength and therefore stability of tunnel valleys sides would have been strongly modulated by variations in basal thermal regime, substrate properties and water flow during glaciation. Using this idea, we propose three theories that could produce these variations in width, and which we hope will motivate physical modelling studies (Fig. 15). Firstly, the variations in tunnel valley width may be a consequence of the very flat beds on which they form (Fig. 14). Water flow in such a landscape will be very sensitive to small changes in bed relief and variations in discharge. Coupled with sluggish water flow due to the low hydraulic gradients, we therefore envisage the tunnel valleys as a series of interconnected swampy regions (Fig. 15a). This is analogous to lakes and or swampy ground connected by overspill channels, or wide flood plains comprising dynamic river channels observed in fluvial systems flowing across similarly flat landscapes. Secondly, a basal thermal regime consisting of a mosaic of cold- and warm-based sediment patches (e.g. Kleman & Glasser, 2007) would locally influence how easily widening could happen (Fig. 15b). Frozen patches would inhibit channel formation and may even result in ponding of meltwater, while warm based patches would be more susceptible to erosion. Thirdly, as discharge increases the conduit can enlarge, either by eroding into the bed (forming tunnel valleys), melting up into the ice (R-channel) or both together (see Fowler, 2011) (Fig. 15c). What happens will vary depending upon, for example, the effective pressure, ice viscosity and sediment stiffness. Consequently, the manifestation of an increase in discharge on the bed imprint is likely to vary spatially and temporally depending on the competition between sediment erosion and the melting of...
ice (e.g. Livingstone et al., 2016). This theory may therefore explain the fragmentation of some tunnel valleys into multiple segments (Fig. 7e,f).

5.3 Landform associations

5.3.1 Relative timing of tunnel valley formation

Cross-cutting relationships between moraines, outwash fans, and tunnel valleys have enabled their relative timing of formation to be used to build a history of formation (Figs. 10, 16). If a tunnel valley cuts through moraine positions, formation must have occurred during or after the moraine was deposited. These tunnel valleys, and those interrupted by outwash fans mid-way along their length, must therefore have been used as a drainage route either repeatedly or over a long duration during retreat (see Fig. 16b). Conversely, tunnel valleys that are cross-cut by recessional moraines were abandoned as ice retreated. In Fig. 16b these tunnel valleys correspond to the age of a single moraine position, and may have been eroded during a singular ‘event’ (i.e. outburst of a sub- or supra-glacial lake) or been abandoned due to a switch in drainage configuration or supply.

5.3.2 Moraines

The close link between tunnel valley networks and moraines (Figs. 3, 10; and see also Attig et al., 1989; Mooers, 1989; Patterson, 1997; Smed, 1988; Johnson, 1999; Cutler et al., 2002; Jørgensen and Sandersen, 2006) suggests formation and growth is intimately associated with pauses or slow-downs in ice retreat or ice advances and that meltwater drained to the ice margin. The implication is that tunnel valley formation requires a relatively stable ice-sheet configuration to allow headward growth or recharge of source storage areas. It also provides further support for the role of permafrost in tunnel valley formation given that rapid retreat will reduce the width of the frozen toe and consequently reduce the efficacy for water storage. However, whether a reconfiguration of the subglacial hydrological regime via the development of tunnel valleys behind ice margins (moraines) can influence ice retreat, for example causing the observed staccato jumps between still-stands (Fig. 16a), remains an open question.

Regularly spaced, low relief transverse ridges (e.g. Fig. 10b), termed washboard or corrugation moraine, have been interpreted as both (annual) end moraine deposits and as subglacial crevasse fill (Kemmis et al., 1981; Stewart et al., 1988; Patterson, 1997; Jennings, 2006; Cline et al., 2015; Ankersjerne et al., 2015). The deflection of transverse ridges towards the long axis of tunnel valleys (e.g. Fig. 10b), and buried sand and gravel deposits (see Stewart et al., 1988; Cline et al., 2015), indicates a temporal and possibly genetic relationship. One interpretation is that lower water and pore water pressures in tunnel valley and glaciofluvial deposits respectively, result in slower local ice velocities that cause the pattern of crevasses and thus ridges to be deflected (see Cline et al., 2015).
However, high pressure discharges have also been inferred from coarse-grained outwash fans deposited in front of tunnel valleys (Section 5.3.4, e.g. Cutler et al., 2002; Jørgensen and Sandersen, 2006). There may therefore have been multiple modes of meltwater drainage down tunnel valleys; predominantly low pressure drainage interrupted by episodic high pressure outbursts.

5.3.3 Hill-hole pairs

The formation of tunnel valleys up-glacier from hill-hole-pairs of similar width (Fig. 12) suggests a temporal relationship. Hill-hole-pair formation is believed to require the ice to be strongly coupled to the bed so that it can exert sufficient shear stress to produce failure (Bluemle and Clayton, 1984; Aber et al., 1989). Thus, either the hill-hole pair was produced first, and the tunnel valley grew headward out of the ‘hole’, or once drainage through the tunnel valley had waned, ice re-coupled strongly to the bed and the downstream termination of the valley became the focus of large shear stresses that resulted in failure and formation of the hill-hole pair. We suggest the former is more likely as the tunnel valleys do not terminate at moraine positions as is typical elsewhere, while small channels and eskers emanating from and diverging around the hills appear to record the down-glacier leakage of pressurised water around the obstruction (Fig. 12b). If true, these tunnel valleys appear to be unique in having initiated up-glacier from the margin. The formation of a hill-hole pair may therefore have facilitated tunnel valley erosion by providing a pathway for water through a frozen toe.

5.3.4 Outwash Fans

Outwash fans occur at the down-glacier end of at least 10% of the tunnel valleys in our study area (e.g. Fig. 10b), and are particularly common along the margins of the Green Bay, Michigan and Langlade lobes (Attig et al., 1989; Clayton et al., 1999; Cutler et al., 2002; Fisher and Taylor, 2002). The fan sediments at the margin of the Green Bay Lobe include well-rounded pebbles and boulders up to 2 m diameter (Cutler et al., 2002), similar to accumulations documented in-front of European tunnel valleys (Piotrowski, 1994; Jørgensen and Sandersen, 2006; Lesemann et al., 2014). The coarse-grained sediments indicate high-energy discharges and or highly pressured subglacial meltwater flow through the tunnel valleys. Cutler et al. (2002) suggested there was at least one large outburst flood just before the termination of glaciofluvial activity through each tunnel valley. These high-energy floods may have been responsible for cutting the valley itself, or the valley could have acted as a preferential drainage route upon tapping into a water reservoir.

5.3.5 Giant Current Ripples

The occurrence of giant current ripples stretching across the whole width of a tunnel valley implies a large sub- or supra-glacial lake outburst event (e.g. Bretz et al., 1956; Carling, 1996; Rudoy, 2005). This is further supported by the circular incision at the southern end of the tunnel valley (Fig. 13).
which is similar in form to large potholes generated by macroturbulent eddies. The flood could have
cut this particular tunnel valley or the valley pre-existed and became the route of a subglacial flood
which completely filled it, further modifying and enlarging the valley (Bretz et al., 1956; Carling,
1996; Rudoy, 2005). The unique occurrence of this landform suggests that large floods were rare or
the landform signature rarely preserved.

5.4 Implications for the formation of tunnel valleys

Based on our large-scale analysis of the morphological properties of tunnel valleys and associated
bedform along the southern portion of the Laurentide Ice Sheet we are able to provide some new
insights into their formation. The importance of ice geometry (Fig. 3) and the semi-regular spacing of
individual tunnel valley networks (Fig. 4), implies a stable, self-organising basal hydrological system
modulated by bed transmissivity, meltwater discharge and the hydraulic potential gradient
(Piotrowski, 1997; Boulton et al., 2007a,b, 2009; Hewitt, 2011). While some tunnel valleys appear to
have been short-lived, either as the preserved signature of a single event or because they were
abandoned due to changes in melt delivery or ice retreat, it is inconceivable that an entire network
pattern was formed during one catastrophic flood (e.g. Shaw, 2002) as many of the valleys are found
to have formed incrementally (also see Mooers, 1989), remaining active and relatively stable over a
long period of time (Fig. 16).

Recurrent outburst of stored water responsible for incremental incision of whole networks is
appealing where tunnel valleys converge towards up-glacier basins (e.g. Superior and Langlade –
Figs. 2a,c, 3) where one could infer that subglacial lakes periodically grew and drained (Evatt et al.,
2006). However, many of the networks are very broad (>60 km across) and the tunnel valleys
relatively parallel (e.g. Green Bay and eastern Superior – Fig. 3). To produce these networks would
require lakes many tens or even hundreds of kms wide. This is difficult to reconcile with mean
(<1km²) and maximum supraglacial lake areas (up to ~150 km² – which equates to a diameter of ~14
km if a circular lake is assumed) on the surface of the present-day Greenland Ice Sheet (e.g. Leeson et
al., 2013). Moreover, while very large subglacial lakes do exist beneath the Antarctic Ice Sheet
(Wright and Siegert, 2011, e.g. Lake Vostok, >250 km long by ~80 km wide) and are theorised to
have existed in Hudson Bay and the Great Lake Basins (e.g. Shoemaker, 1991, 1999), they have
never been predicted by modelling or identified in the geological record (e.g. Livingstone et al.,
2013).

Despite the lack of support for a mega-flood genesis of whole tunnel valley networks, drainage of
stored water down individual valleys almost certainly did happen. Not all tunnel valleys formed in
networks or were incised time-transgressively up-glacier (Figs. 3, 16), and the simplest explanation
for the formation of fans containing boulders (Figs. 2, 10b) (e.g. Piotrowski, 1994; Cutler et al., 2002;
Lesemann et al., 2014) and for giant current-ripples (Fig. 12) is high discharge (possibly bank-full)
events. Indeed, periodic higher energy or pressurised meltwater events (e.g. during penetration of surface meltwater to the bed during summer months) were probably necessary to prevent armouring of the valley sides by coarse sediment, while bedrock tunnel valleys are difficult to reconcile solely by gradual formation. We therefore contend that large drainage events from sub- and supra-glacial lakes, and by injections of surface meltwater down moulins did occur, contributing to the formation of tunnel valleys either by eroding new valleys or enlarging existing ones. However, our data suggests they were probably not the primary mechanism by which tunnel valleys formed. Firstly, the decline in tunnel valley incidence away from LGM margin positions (Fig. 14) is inconsistent with increasing contributions of surface melt in an ameliorating climate. Secondly, their typical length distribution (Fig. 5a) is an order of magnitude less than the distance up-glacier (tens to hundreds of km) that supraglacial and subglacial lakes are commonly documented in Greenland and Antarctica (e.g. Selmes et al., 2011; Wright and Siegert, 2011).

We suggest that the majority of tunnel valleys along the southern sector of the Laurentide Ice Sheet were initiated at the ice margin and then typically (although not exclusively) eroded gradually up-glacier. Tunnel valley length and width display log-normal distributions and are positively correlated, indicative of a growing phenomenon (cf. Fowler et al., 2013; Hillier et al., 2013). Their strong association with moraine positions (Fig. 3) suggests that formation is time dependent (i.e. they require time to grow), while cross-cutting relationships (Fig. 16) demonstrates that many of the features remained active for extended periods. Thus, when retreat is slow or a stable position is reached (allowing formation of a moraine), tunnel valleys have time to grow up-glacier and to widen and deepen as more water is discharged through them (Fig. 17a). A more unstable/rapid ice-retreat will limit the time for growth (headward and lateral) or may even produce a segmented tunnel valley if retreat overtakes headwards incision (Fig. 17b). Indeed, the James and Des Moines ice lobes that are thought to have rapidly surged to and then retreated from their maximum positions (Clayton and Moran, 1982; Clayton et al., 1985) are relatively devoid of well-organised tunnel valley networks compared to other ice lobes, such as Superior, that retreated more slowly (Dyke, 2004). We argue that growth was not a function of conditions associated with the size of a stored water body and the magnitude and frequency of its drainage because immature (smaller) tunnel valleys are also found to preferentially terminate at ice-margin positions (e.g. southern margin of Green Bay Lobe, Fig. 3). Hence, growth likely initiated and proceeded up-glacier from the ice margin rather than down-glacier from a stored water body, and there is some evidence for this, including the presence of amphitheatre-shaped tunnel valley heads (e.g. Onda, 1994; Abrams et al., 2009; Petroff, 2011) and the growth of valleys out of hill-hole-pairs (Fig. 11).

6. Summary and Conclusions
There have traditionally been two main paradigms to explain the formation of tunnel valleys: (1) outburst formation by rapid drainage of sub- and/or supraglacially stored meltwater; and (2) gradual formation by headward sapping in low pressure subglacial channels (Fig. 1) (cf. Ó Cofaigh, 1996; Kehew et al., 2012; van der Vegt et al., 2012). To investigate these two models we undertook a large-scale mapping campaign to characterise the distribution and morphology of >1900 tunnel valleys and associated bedforms on the bed of the former Laurentide Ice Sheet.

Given our previous work on subglacial lakes beneath the Laurentide Ice Sheet (Livingstone et al., 2013), we specifically explored tunnel valleys with an expectation that they might link with predicted lake locations and be the geomorphological record of outburst floods. However, to the contrary the morphological evidence suggests gradual formation, with some contributions from large drainages of stored water (Fig. 18). In particular, our findings indicate that tunnel valleys comprise well-organised networks of semi-regularly spaced (1.9-9.1 km) valleys that formed incrementally during ice retreat. This pattern is strongly controlled by ice geometry and basal properties (e.g. permafrost, flat bed and conduit erosion), and this is a strong argument for a self-organising hydrological network influenced by local conditions. Second, tunnel valleys preferentially terminate at moraines (irrespective of their size), which suggests that growth was initiated at and then progressed headwards from stable ice-margin positions. The concept of a growing phenomenon is further supported by log-normally distributed valley morphologies, the positive correlation between length and width, their initiation and growth out of hill-hole-pairs and the existence of amphitheatre-shaped valley heads. Although we favour gradual headward formation as the primary process, our results also show examples where outburst of supraglacial and or subglacial lakes have incised and/or drained down valleys. Evidence includes, giant current ripples and outwash fans with large boulders (Cutler et al., 2002), and some valleys were only occupied for brief periods during deglaciation suggestive perhaps of a short-lived event. Indeed, cross-cutting relationships demonstrate a time-transgressive origin for many tunnel valleys, with individual networks forming within the same time frame but individual valleys evolving over different spans involving multiple discrete flow events.

Many of our observations are consistent with previous findings (e.g. Kehew et al., 2012 and references therein) and we are not the first to suggest a polygenetic origin (e.g. Hooke and Jennings, 2006). However, whilst geomorphological and sedimentological investigations in certain areas have generally advocated either an outburst or gradual genesis for tunnel valleys (Fig. 1), when their morphology, distribution and association with other glacial bedforms are considered at a regional-scale it suggests that both processes occurred (Fig. 18).

At the ice-sheet scale, we find most tunnel valleys occur on the flat portions of palaeo-ice sheet beds, where subglacial water flow would have been largely unconstrained by topography. It is on these portions of the bed, where ice-geometry is the main control, that subglacial water becomes organised
down relatively stable and regularly-spaced drainage corridors (tunnel valleys). Once a tunnel valley has been initiated, it could provide a low pressure 'release valve' (i.e. generate a local hydraulic gradient) to evacuate basal water flowing slowly through water saturated sediments and swampy ground (after Kyrke-Smith and Fowler, 2014) in areas of the bed characterised by low hydraulic gradients, and also as a routeway for large injections of surface or stored water. These drainage corridors provide an effective means of transporting sediment and water from under the ice sheet and may thus have acted to increase basal traction across the bed and slow-down ice flow during deglaciation.

**Author contributions**

SJL and CDC designed the project. SJL generated the data on the tunnel valleys and other glacial bedforms. Both authors contributed to the analyses and interpretations of the data. SJL wrote the manuscript with input from CDC.

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Figure 1: Cartoons depicting the two main models for tunnel valley formation: A. Outburst floods from supraglacial and/or subglacially stored water; and B. Gradual headward growth by sapping.
Figure 2: Examples of mapped valleys and the assignment of confidence levels (1 = high confidence to 3 = low confidence) along the southern sector of the former Laurentide Ice Sheet. Valleys in panels A (Superior), B (Saginaw), C (North Dakota) and D (Green Bay) are assigned a confidence of 1. The relict valleys contain eskers, are parallel and relatively straight, and do not trend along the regional slope. In panels A, C and D the tunnel valley networks terminates at a moraine position. The large
valley in panel E (Superior) is assigned a confidence level of 3 as it does not contain any subglacial bedforms and exhibits a gradual and consistent change in bed slope consistent with a proglacial spillway. However, the smaller NW-SE valleys that bisects is given a confidence of 2 as they have undulating thalwegs that cut across moraines. The dendritic valley network in panel F (North Dakota) is given a confidence of 3 as it is not associated with any subglacial bedforms and has a consistent bed slope indicating water flow towards the south. A braided channel morphology and a widening reach towards the south allows us to interpret this valley system as a proglacial spillway (fed by tunnel valleys emanating from under the ice to the north).

Figure 3: Distribution of mapped tunnel valleys and moraines along the southern sector of the Laurentide Ice Sheet. Likely subglacial lake locations are predictions from Livingstone et al., (2013). The Last Glacial Maximum extent is from Dyke et al., (2004) and moraines are from Fullerton et al. (2003).
Figure 4: Frequency histogram of the spacing of 966 tunnel valleys from 24 discrete networks across the southern sector of the former Laurentide Ice Sheet.

Figure 5: Frequency histogram of tunnel valley length and width (for confidence levels 1 and 2). Line is the log-normal distribution for comparison. Width values were extracted at 1 km intervals along the centre-line of each tunnel valley.
Figure 6: Relationship between tunnel valley length and average width (for single thread valleys with a confidence level of 1 and 2, N=1135). Note, there is a tendency for longer tunnel valleys to be wider.
Figure 7: Examples of tunnel valley morphology of tunnel valleys with a confidence of 1 or 2. A. Superior Lobe (note the amphitheatre heads of some valleys); B. Wadena Lobe (note the large downstream changes in tunnel valley width); C. Langlade Lobe; D. Saginaw Lobe; and E. Wadena (note the parallel valleys) and F Huron-Erie (note the abrupt start and end points of the tunnel valleys and parallel organisation).
Figure 8: Along-valley plots highlighting tunnel valley width variations. A. Saginaw Lobe; B. North Dakota; C. Green Bay Lobe; D. Superior Lobe; and E. Wadena Lobe. F. Cartoon showing the expected relationship between width and distance downstream for a fluvial river (Leopold and Maddock, 1953; Leopold et al., 1964) and single flood event (Lamb and Fonstad, 2010). Note that the measured tunnel valley width variations conform to neither of these expectations, but instead show variations in width greatly exceeding any possible systematic trends.
Figure 9: Scatter plot compiled to investigate if downstream variation in channel width was controlled by variations in downstream slope gradient (see text for details). That the data are centred on zero and spread fairly evenly around this demonstrates that there is no systematic relationship between elevation gradient (i.e., whether it is a normal or reverse gradient slope) and width (i.e., whether the tunnel valley is narrowing or widening).
Figure 1: The varied cross-cutting associations between moraines, outwash fans and tunnel valleys in: A. North Dakota – note how the washboard moraines curve up-glacier towards the tunnel valleys; and B. Wisconsin (Chippewa Lobe).
Figure 11: Hill-hole-pairs in North Dakota and their association with tunnel valleys. 

A. Note the esker downstream of the hill, which trends into an aligned tunnel valley segment.

B. Note the secondary meltwater channels and eskers that diverge around the hill.
Figure 12: Giant Current Ripples spanning the width of a shallow tunnel valley that is cut into an obliquely-oriented drumlin field (water flow to the south). These sinusoidal bedforms are interpreted as giant current ripples, which formed during a large subglacial flood. Note the undulating thalwegs and esker in the valley that indicates subglacial deposition, and the circular incision (with a remnant island in its centre) in the south of the valley that may have formed by a large eddy during high-energy turbulent flow.
Figure 13: Idealised ice lobe, hydraulic potential contours (dotted lines) and drainage routes. Note how this predicts that with uniform upstream basal melting that the resultant water paths diverge down the lobe axis and away from the terminus (yielding low water delivery here) and converge in interlobate regions (high water delivery).

Figure 14: Currently known Northern Hemisphere distribution of tunnel valleys that have been attributed to the last glaciation. The opaque blue shading is the Last Glacial Maximum ice sheet.
distribution. Black lines are the mapped tunnel valleys from Fig. 3 and black boxes are where tunnel valleys have been identified.

**Figure 15:** Cartoons showing three theories to explain the downstream variation in tunnel valley width. A. Swampy ground (blue stipples) and channels (blue lines) associated with water flow across very flat ground. In such a flat landscape tunnel valleys are able to easily expand laterally, in response to small changes in water flux, and there is little impetus for rapid vertical erosion due to shallow hydraulic gradients. B. Tunnel valley formation is modulated by the basal thermal regime (modified from Hughes, 1995). Channels are able to develop more easily across warm sediment patches, and the mosaic of cold and warm sediment patches results in variations in width. C. Undulatory conduit erosion. In this theory the width of the channel eroded into sediment depends upon the competition between erosion down into the sediment (canals) vs. melting up into the ice (R-channel) (see Fowler, 2011; Livingstone et al. Sub). Note that each of the conduits (i-iii) have the roughly the same area, but that in (ii) no channel forms and in (iii) the channel width is roughly half that of (i).
Figure 16: Using cross-cutting relationships to reconstruct tunnel valley evolution during ice margin retreat. **A.** Mapping of tunnel valleys and associated glacial bedforms in Wisconsin (Chippewa Lobe) (from Fig. 11B). **B.** Reconstructed history of valley formation behind a back-stepping ice margin. Note that some valleys were long-lived during deglaciation and some abandoned shortly after their formation. The relative age relations help explain the variation in lengths between long continuous tunnel valleys and those comprising short fragments.
Figure 17: Cartoon demonstrating the dependence of tunnel valley evolution (by headward growth) on ice margin retreat rate. **A.** If headward growth of a tunnel valley is faster than the rate of ice retreat, the valley will be able to extend continuously up-glacier and its length will only be limited by water supply and hydraulic properties of the bed. **B.** If however, headward growth of a tunnel valley is slower than the rate of ice retreat the valley is likely to be discontinuous, only being able to form and extend up-ice during slow-downs or pauses in retreat.
Figure 18: For the southern Laurentide region we consider gradual headward erosion as the usual mechanism, but with some floods down selected valleys – note the potential for stored water to cut their own valleys (e.g. supraglacial lake drainage example) or to drain along pre-existing corridors that may have tapped into a reservoir (e.g. subglacial lake example).