

## ***Interactive comment on “Scaling and similarity of a stream-power incision and linear diffusion landscape evolution model” by Nikos Theodoratos et al.***

**G.E. Tucker (Referee)**

gtucker@colorado.edu

Received and published: 14 May 2018

Overview:

This paper presents an insightful analysis of the simplest form of governing equation for drainage basin evolution. The authors present a dimensional analysis of the equation, showing that by treating horizontal and vertical scales separately, and by using the parameters themselves as the basis for normalization, one arrives at a simple and elegant dimensionless rendering—which contains important lessons about similarity. From the analysis emerge three characteristic scales, representing horizontal length, vertical distance (height), and time, respectively. And from this scale analysis emerge

C1

valuable insights into a number of different aspects of basin and landscape evolution, such as: what sets the scale of transition between hillslopes and valleys? How do the efficiencies of hillslope transport and channel incision influence rates of drainage-divide migration? How are process parameters, which represent relative efficiencies of a geomorphic processes, manifested in quantitative terrain metrics? This is a theoretical exercise that shows very nicely how we can usefully think about the implications of the "stream power plus diffusion" model of landscape evolution: what are the key scales to consider, and how the parameters relate to one another. And although this ground has been covered before to some extent (most notably in the work of Garry Willgoose and colleagues from 1991), the present paper represents a valuable advance because it shows that one can capture the fundamental scaling of the system purely with reference to the process parameters.

I really appreciate that the authors took the time to craft an impeccable manuscript. The writing is lucid, the scope is thorough, and the arguments are well articulated. In fact, I've rarely seen such a flawless presentation in a first-time submission. Well done!

The decision to focus on the special case of  $m=1/2$ ,  $n=1$ , while detailing the more general case in an appendix, strikes me as a sensible approach. The special case turns out to be beautifully simple. The general case is more complicated, and if they had tried to cover it in the main text the result would have been harder to penetrate. Yet they show that the fundamental insights derived from the simpler case usually hold for the more general one as well.

Specific comments keyed to text by page / line number:

1/8 Just an observation that we have a bit of a terminology problem as a community, which is illustrated here with the use of "Landscape Evolution Model" to refer to a set of governing equations. The word "model" is variously used to describe qualitative concepts, equation sets, numerical approximation methods and algorithms that represent certain processes, and particular computer programs that implement those numerics.

C2

Of course, I don't expect the authors to solve this problem! But given that "LEM" has been used in the past to describe numerical models, you might consider specifying "widely used equation set for landscape evolution models (LEMs)..."

2/ 28 suggest "describes" rather than "simulates" (e.g., it would seem odd to say that mg "simulates" the force of gravity on an object at earth's surface, though if you calculated in a computer program that graphically depicts a falling rock, that might reasonably be called a simulation)

3/8 "in keeping with the stream power law": there are some devils in the details here; maybe simplest to summarize as "in keeping with the stream power law and the common assumption that discharge per unit width scales with the square root of drainage area"

3/17-18 this is just a minor terminology issue, but for what it's worth I think of soil creep as a general transport phenomenon that can be caused by the various processes listed, and that is often described quantitatively using a diffusion model. To say "diffusive process" mixes the process and the model thereof. To list soil creep as a process comparable to bioturbation somehow mixes a possible causative agent (bioturbation) with the phenomenon that results (soil creep). Consider re-wording.

3/23-25 nice job being clear about baselevel lowering being mathematically equivalent to "uplift", which therefore needn't be caused by some kind of odd hillslope-scale vertical tectonics!

4/7-8 I think it would be fair to also cite Willgoose et al. 1991b (WRR, "part 2") here. I think of this work as the first serious introduction of dimensional analysis applied to landscape evolution, and he and his colleagues Ignacio Rodriguez-Iturbe and Rafael Bras deserve credit for leading the way.

eq 16 and following paragraph: beautiful!

6/21 this is an odd result, which seems to neglect the fact that dimensionless drainage

C3

area can in principle range over orders of magnitude—thus, it's possible to have elements of the solution domain in which either the 1st or 2nd term on RHS dominates... which the next paragraph acknowledges.

8/12 why 34 in particular? Was this a systematic experimental design?

8/12-23 it would be helpful to state whether the model was actually run in dimensional form and then the output rescaled (which is probably the case). The sentence "We rescaled..." implies that dimensional grid coordinates were converted into dimensionless equivalents, but it is not explicitly stated whether this rescaling was applied to the inputs (i.e., input grid and  $K$ ,  $D$ ,  $U$ ) or the outputs.

8/25-26 same question arises here: "run on rescaled versions of the same random TIN" is ambiguous because it seems to suggest that you took a dimensional TIN and rendered it dimensionless before conducting each run. I'm guessing that what was actually done was to calculate ahead of time, for each set of parameters, what horizontal and spatial dimensions would produce a size of  $200 \times 400 l_c$  and  $0-0.1 h_c$ , and then set up dimensional model runs using these spatial dimensions. Is that correct? The alternative I suppose is that the inputs were re-scaled, meaning you used units of  $l_c$ ,  $h_c$ , and  $t_c$  instead of meters and years; but if that were the case, the initial grid would be always exactly the same in horizontal point locations, with the only differences being in the vertical scale.

9/1 I like how you varied  $l_c$  and  $h_c$  independently in these illustrations

10/19-28 wonderful insight into the divide migration problem—nice example of how much can be learned from dimensional analysis

section 4.1.1 - Here, interpretations for height scales are offered in terms of the characteristic time scale. It would be useful therefore to suggest an intuitive explanation for what  $t_c$  represents. Given the definition of  $t_c$  as  $1/K$ , one such explanation is that it is the time required for one unit of incision given one unit area-slope quantity. "Unit area-slope

C4

quantity", or "unit steepness", of course includes all cases of  $\sqrt{A} S = 1$  (eg,  $S=1$  at  $A=1\text{m}^2$ ,  $S=0.1$  at  $A=100\text{m}^2$ ,  $S=0.01$  at  $A=10,000\text{m}^2$ ,  $S=0.001$  at  $A=1,000,000\text{m}^2$ ). There's a nice rule of thumb in here: if I did the math right, unit steepness includes a slope of one-in-a-hundred at a drainage area of 1 hectare, or one-in-a-thousand at a drainage area of 1  $\text{km}^2$ .

eq 27: suggest pointing out to readers that this is just a rearrangement of the expected slope-area relationship for channels at steady state—thus, one recaptures this important relation.

fig 6: this is a nice illustration. If I were to change anything, it would simply be to plot  $-h_I$  (i.e., include the minus sign implied by eq 29 in relation to  $h_C$  and  $h_D$ ).

15/10-17 This is a nice testable prediction. I can imagine issues arising related to the identification of drainage area (e.g., depending on the routing scheme used, at high res, many points within a valley could register as low  $A$  even though they are morphologically influenced by channelized flow), but worth trying.

22/16-17 suggest making these separate sentences

C1.2 why was it necessary to generate TINs using Matlab rather than just relying on CHILDS's own TIN generation routines?

---

Interactive comment on Earth Surf. Dynam. Discuss., <https://doi.org/10.5194/esurf-2018-33>, 2018.